Applications of the Integrals

Assertion & Reason Type Questions

Directions: In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true and Reason (R) is false

d. Assertion (A) is false and Reason (R) is true

Q1.

Assertion (A): The area of the region bounded by the curve $y^2 = 4x$ and the line $x = 3$ is $8\sqrt{3}$ sq. units.
Reason (R): If $f(x) \geq 0$ is a continuous function which is defined in the interval $[a, b]$, then the area of the region bounded by the curve $y = f(x)$, X-axis, $x = a$ and $x = b$ is given by

$$\text{Area} = \int_a^b f(x)\,dx = \int_a^b y\,dx$$

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q2.

Assertion (A): The area bounded by the parabola $y^2 = 4ax$ and the lines $x = a$ and $x = 4a$ is $\frac{56a^2}{3}$ sq. units.

Reason (R): Area of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab sq. units.

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

Q3.

Assertion (A): The area enclosed by the curve $|x|+|y|=2$ is 8 units.
Reason (R): $|x|+|y|=2$ represents a square of side length $\sqrt{8}$ units.

Answer : (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

Q4.

Assertion (A): The area bounded by the curve $y=2\cos x$ and the X-axis from $x=0$ to $x=2\pi$ is 8 sq. units.
Reason (R): The area bounded by the curve $y=\sin x$ between $x=0$ and $x=2\pi$ is 2 sq. units.

Answer : (c) Assertion (A) is true and Reason (R) is false

Q5.

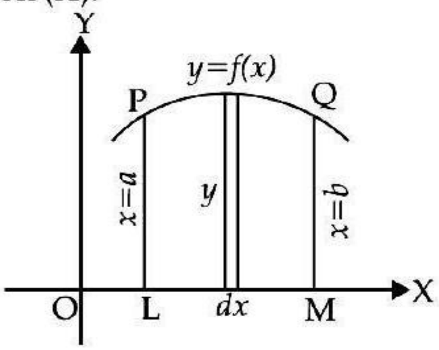
Assertion (A): The area of the region in the first quadrant, bounded by the parabola $y=9x^2$ and the lines $x=0$, $y=1$ and $y=4$ is 14/9 sq. units.
Reason (R): If for $x\in[a,c]$, $f(x)\geq 0$ and for $x\in[c,b]$, $f(x)\leq 0$, where $a<c<b$, then area of region bounded by curve $y=f(x)$, X-axis, $x=a$ and $x=b$ is given by

$$\text{Area}=\int_a^c f(x)\,dx-\int_c^b f(x)\,dx.$$

Answer : (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

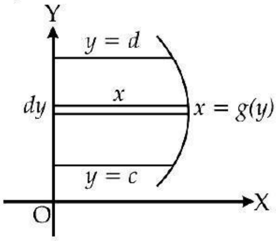
Q6

Assertion (A):



The area of region $PQML = \int_a^b y\,dx = \int_a^b f(x)\,dx$

Reason (R):



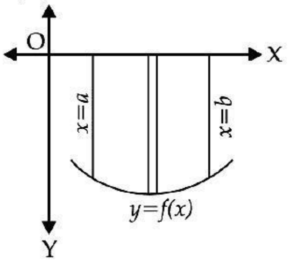
The area A of the region bounded by curve $x = g(y)$, y-axis and the lines $y = c$ and $y = d$ is given by

$$A = \int_c^d x\,dy$$

Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are individually correct.

Assertion (A):



$$\text{Area} = \left|\int_a^b f(x)\,dx\right|$$

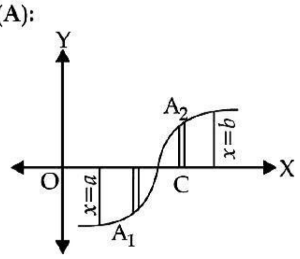
Reason (R): If the curve under consideration lies below x-axis, then $f(x) < 0$ from $x = a$ to $x = b$, the area bounded by the curve $y = f(x)$ and the ordinates $x = a$, $x = b$ and x-axis is negative. But, if the numerical value of the area is to be taken into consideration, then

$$\text{Area} - \left|\int_a^b f(x)\,dx\right|$$

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Assertion (A):



$$\text{Area} = |A_1| + |A_2|$$

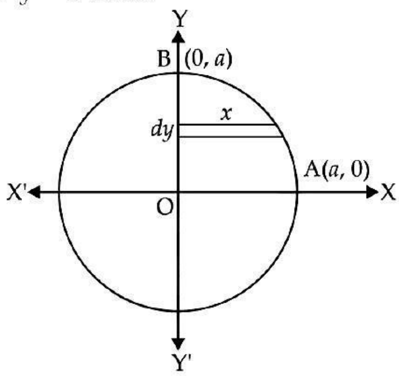
Reason (R): It may happen that some portion of the curve is above x-axis and some portion is below x-axis as shown in the figure. Let A_1 be the area below x-axis and A_2 be the area above the x-axis. Therefore, area bounded by the curve $y = f(x)$, x-axis and the ordinates $x = a$ and $x = b$ is given by

$$\text{Area} = |A_1| + |A_2|$$

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Assertion (A): The area enclosed by the circle $x^2 + y^2 = a^2$ is πa^2.



Reason (R): The area enclosed by the circle

$$= 4\int_0^a x\,dy$$

$$= 4\int_0^a \sqrt{a^2 - y^2}\,dy$$

$$= 4\left[\frac{y}{2}\sqrt{a^2 - y^2} + \frac{a^2}{2}\sin^{-1}\frac{y}{a}\right]_0^a$$

$$= 4\left[\left(\frac{a}{2}\times 0 + \frac{a^2}{2}\sin^{-1}1\right) - 0\right]$$

$$= 4\frac{a^2}{2}\frac{\pi}{2}$$

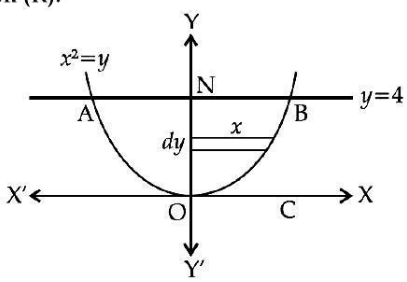
$$= \pi a^2$$

Ans. Option (A) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are correct, Reason (R) is the correct explanation of Assertion (A).

Assertion (A): The area of the region bounded by the curve $y = x^2$ and the line $y = 4$ is $\frac{3}{32}$.

Reason (R):



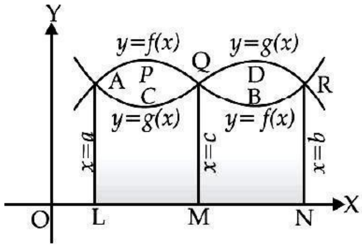
Since the given curve represented by the equation $y = x^2$ is a parabola symmetrical about y-axis only, therefore, from figure, the required area of the region $AOBA$ is given by

$$A = 2\int_0^4 x\,dy$$
$$= 2\int_0^4 \sqrt{y}\,dy$$
$$= 2 \times \frac{2}{3}\left[y^{3/2}\right]_0^4$$
$$= \frac{4}{3} \times 8$$
$$= \frac{32}{3}$$

Ans. Option (D) is correct.

> ***Explanation:*** Assertion (A) is wrong. Reason (R) is the correct solution of Assertion (A).

Assertion (A): If the two curves $y = f(x)$ and $y = g(x)$ intersect at $x = a$, $x = c$ and $x = b$, such that $a < c < b$.

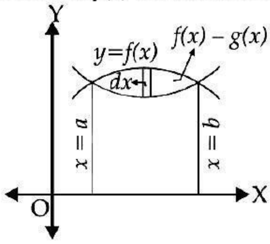


If $f(x) > g(x)$ in $[a, c]$ and $g(x) \le f(x)$ in $[c, b]$, then Area of the regions bounded by the curve

= Area of region $PACQP$ + Area of region $QDRBQ$.

$$= \int_a^c |f(x) - g(x)|\,dx + \int_c^b |g(x) - f(x)|\,dx.$$

Reason (R): Let the two curves by $y = f(x)$ and $y = g(x)$, as shown in the figure. Suppose these curves intersect at $f(x)$ with width dx.



$$\text{Area} = \int_a^b [f(x) - g(x)]dx$$
$$= \int_a^b f(x)dx - \int_a^b g(x)dx$$

= Area bounded by the curve $\{y = f(x)\}$

–Area bounded by the curve $\{y = g(x)\}$,

where $f(x) > g(x)$.

Ans. Option (B) is correct.

> ***Explanation:*** Assertion (A) and Reason (R) both are individually correct.